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Three Dimensional Flow of Nanofluid over an Exponentially Stretching Sheet with Chemical Reaction and Activation Energy

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Abstract

Motive of this paper is to examine the effects of steady three dimensional MHD free convective boundary layer flow of nanofluid over a bi-directional exponentially stretching sheet with chemical reaction and activation energy. The well-dispersed (metallic) nanoparticle at low-volume fractions in liquids is known as nanofluids. They may enhance the mixture's properties and thermal conductivity over the base fluid values. An induced magnetic field can be used to control the movement of an electrically conducting fluid in micro-scale systems used for the transportation of fluids. The mathematical model is framed in such a way that the effects of Brownian motion and thermophoretic diffusion of nanoparticles are considered. It is assumed that the temperature and nanoparticle volume fraction at the sheet are also disseminated exponentially.The solutions for the governing equations are obtained by employing finite difference method. The effects of various controlling parameters on the dimensionless velocity, temperature and nanoparticle volume fraction profiles are discussed graphically.

Keywords: *MHD, Nanofluid, Exponentially Stretching Sheet, Binary chemical reaction, Activation energy.*

1. Introduction

Heat transfer mechanism has an important role in many engineering and industrial fields because cooling and heating processes are involved in such fields. An increase in heat transfer rate is quite essential. It reduces the process time of work and length of the work life of equipment. Various methods are proposed in the past to increase the heat transfer efficiency rate. Some methods involve extended surfaces.

In all processes involving energy-efficient heat transfer, the thermal conductivity of the fluids is one of the basic properties taken in to account in designing and controlling the process. However, the traditional pure liquid heat transfer medium has a low thermal conductivity, which limits the heat transfer enhancement. As a result, it is essential to prepare a higher thermal conductivity and more efficient heat transfer medium. Scientific breakthrough has been made by Choi in 1995 when for the first time he introduced the term "Nanofluids". Nanofluids are a new division of heat transfer fluids with significantly higher conductivities. These fluids are developed by suspending nanoscale metallic or nonmetallic particles in the base fluid. Now a day's studies on nanofluids has been receiving a lot of attention worldwide due to its exclusive properties which make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines.

In physics and fluid mechanics, a boundary layer is an important concept and refers to the layer of fluid in the immediate vicinity of a bounding surface where the effects of viscosity are noteworthy. The deduction of the boundary layer equations was one of the most important advances in fluid dynamics. The flow due to an impulsive motion of a moving extensible surface due to boundary layer is involved in various industrial and technological applications such as metal and polymer extrusion, aerodynamic extrusion of plastic sheets, glass blowing, crystal growing, paper production etc. Sakiadis [1] has initiated with the pioneering work on the boundary layer flow over a moving plate in a stationary ambient fluid. Crane [2] studied the Sakiadis problem for a stretching sheet and obtained a closed form exact solution for the velocity distribution. The flow of elasto—viscous fluid bounded by a stretching sheet was reported by Raja gopal et al. [3]. He concluded that rate of cooling of the extruded polymer sheet is larger in visco elastic fluid when compared with the viscous fluid. Lawrence and Rao [4] extended this work for heat transfer characteristics. Based on these fundamental studies, stretching sheet problem in two- and three-dimensional flows has been comprehensively analyzed by the researchers Grubka and Bobba [5], Banks [6], Chen and Char [7], Ali [8], Pop and Na [9] reported on MHD flow over a stretching permeable surface.

Magyari and Keller [10],Liao [11], Liao [12], Xu et al. [13], Sajid et al. [14], Liu and Anderson [15] presented their studies based on boundary layer flow of the fluid over a stretching sheet and the analysis was further carried out by Xu and Liao [16], Hayat et al. [17] and Junaid et al. [18]. These studies were only restrained to the flow over linearly stretching surfaces. However, in industrial applications mentioned above, the velocity of the extruded sheet may not be necessarily linear. Keeping this in view, Magyari and Keller [19] studied the two-dimensional viscous flow over caused by an exponentially stretching sheet. In this study it is assumed that the surface heat transfer was also exponentially distributed. Khan and Sanjayanand [20] examined heat transfer of viscoelastic boundary layer flow over an exponentially stretching sheet and obtained an approximate analytical solution. Homotopy analytic solutions for two-dimensional flow over an exponentially stretching sheet with thermal radiation were reported by Sajid and Hayat [21]. At present Liu et al. [22] provided an excellent numerical study on the three-dimensional viscous flow past an exponentially stretching sheet.

The boundary layer flow on a continuously moving surface is the flow wherein a plane surface is continuously drawn from a wall into a quiescent fluid. Such a type of flow occurs in a number of mechanical and technical processes such as (a) melt spinning, (b) continuous casting, (c) cooling of metallic sheets, (d) glass blowing, (e) drawing of glass sheets through a quiescent fluid, and (f) extrusion of films and plates. In the process of drawing of artificial fibres, the polymer emerges from an orifice with the speed which increases from zero at the orifice up to a certain value after which it remains constant. The moving fibre produces a boundary layer in the medium surrounding the fibre. In the case of cooling of continuous strips or filaments by drawing them through a quiescent fluid, such strips are stretched. In all these processes, the stresses induced in the extruded sheet and the heat transfer rate during the formation control the strength and the quality of the produced material as well as the production rate and cost. Thus a detailed knowledge of velocity, temperature and concentration distributions in this layer is very important in controlling the rate so as to obtain the final product of desired characteristics. The boundary layer formed due to the continuously moving surface is different from that in the Blasius flow past a flat plate due to the entrainment of the ambient fluid.

The boundary layer flow over a continuously stretching surface is commonly encountered in various industrial and engineering processes such as materials manufactured by extrusion of plastic sheets and materials traveling between a windup roll and feed roll. Numerous studies have been done on the two-dimensional boundary layer flow over a stretching surface. Not much attention is paid to three-dimensional boundary layer flow induced by stretching surface.

Radiation effects on the boundary layer flow of Jeffrey fluid above an exponentially stretching sheet were analyzed by Nadeem et al. [23].Traditional heat transfer fluids such as water, ethylene-glycol, engine oil, lubricants etc. possess limited heat transfer capabilities due to their low thermal conductivity and are inadequate to meet the modern cooling requirements. On the other hand metals possess extremely higher thermal conductivity in contrast to the conventional heat transfer fluids. Masuda et al. [24] initially pointed out that viscosity and thermal conductivity of the liquids can be altered by using nanoparticles which are made up of metals, oxides, carbides and carbon nanotubes in the base fluids. Choi and Eastman [25] have observed the unexpected increase in the thermal conductivity through the dispersion of nanoparticles in the base fluid. The improved thermal behavior of nanofluids has vital importance in many industrial fields including power generation, transportation, micro-manufacturing, micro-electronics, pharmaceutical processes, thermal therapy for cancer treatment, chemical and metallurgical sectors, air-conditioning etc.The application of nanofluids as coolants in automobiles would allow for enhanced size and positioning of the radiators and hence this will require less energy for overcoming resistance on the road. Due to a considerable improvement in vehicle aerodynamics, there is an elevated demand for braking systems with higher and more efficient heat dissipation mechanisms and properties such as brake nanofluid. Researchers also suggested the use nanofluid based solar collectors for optimal absorption of solar radiations (see Trieb and Nitsch [26], Otanicar et al. [27] and Ladjevardi et al. [28]. The magnetic nanoparticles are important in medicine, construction of loud speakers, sink float separation, cancer therapy and tumor analysis. The thermal behaviour of magnetic nanoparticles is also tunable through the variations in the magnetic field strength. It is also pointed out recently that magnetic nanoparticles are injected into the blood vessels nearest to the cancerous tissues [29].Based on the above mentioned applications, Buongiorno [30] analyzed the convective transport in nanofluids and concluded the Brownian motion and thermophoresis as the most important mechanisms for the abnormal heat transfer enrichment. Natural convective boundary layer flows of nanofluids past a vertical flat plate were explored by Kuznetsov and Nield [31] and Nield and Kuznetsov [32]. They derived the governing equations for nanofluid flow through Buongiorno's model. It is also evident that rate of cooling of the extruded polymer sheet can be improved by using nanofluids. In view of this the conventional problem of two dimensional flows over a linearly stretching sheet in the presence of nanoparticles was conducted by Khan and Pop [33]. Later Makinde and Aziz [34] reconsidered the work of Khan and Pop [33] by taking convective boundary condition. Mustafa et al. [35] presented an analytic solution for stagnation-point flow of a nanofluid by using homotopy analysis method .Mustafa et al. [36, 37] applied HAM method to explore the two-dimensional exponentially stretching sheet problem for nanofluids. Rana and Bhargava [38] analyzed the flow of nanofluid over a nonlinearly stretching sheet by finite element method.

Bég et al. [39] numerically investigated the unsteady MHD mixed convective boundary layer flow of a nanofluid induced by an exponentially stretching sheet embedded in a porous medium. Numerical solutions for a nanofluid past a stretching circular cylinder with non-uniform heat source was reported by Rasekh et al. [40]. Uddin et al. [41] presened the steady two-dimensional MHD free convective boundary layer flow of an electrically conducting nanofluid past a vertical flat plate with the boundary condition based on Newtonian heating. Ashorynejad et al. [42] investigated nanofluid flow over stretching cylinder in the presence of magnetic field. Mustafa et al. [43] studied the unsteady boundary layer flow of nanofluid past an impulsively stretching sheet by HAM method. Exact analytic solutions of unsteady convective heat transfer problem for various nanofluids have been obtained by Turkyilmazoglu [44]. Numerical solution for non-linear radiation heat transfer problem in nanofluids with an application to solar energy was computed by Mushtaq et al. [45]. Flow of nanofluid due to a rotating disk was stated by Turkyilmazoglu [46]. The effects of magnetic field on the flow of Cu-water nanofluid were discussed by Sheikholeslami et al.[47]. Safei et.al.[48] discussed the heat transfer enhancement in nanofluids using nanotubes in forward-facing contracting channel. Malvandi and Ganji [49] examined the flow of water or aluminum based nanofluids through circular channel with magnetic field. Mixed convection flow past a vertical micro-channel was addressed by Malvandi and Ganji [50]. In another paper, Malvandi and Ganji [51] forced convection flow of nanofluid in a cooled plate micro-channel was considered. Karimipour et.al.[52] used lattice Boltzmann method to discuss the mixed convection of cu/water nanofluid inside an inclined lid driven cavity. The three-dimensional flow of nanofluid over an exponentially stretching sheet is not considered by the researchers. This current work is undertaken to extend the flow analysis of Liu et al. [23] for nanofluid (by incorporating the combined effects of Brownian motion and thermophoresis). Although we employ a similarity approach to non dimensionalize the problem but since coordinates x and y could not be eliminated from the dimensionless equations, the solutions are locally similar. Such kind of solutions can be used to see the variation of parameters at fixed location above the stretching sheet (which is coincident with the xy—plane). Current studies concerned with the local similarity solutions of the boundary layer equations can be found in refs. [53–59]. The equations are solved for the numerical solutions by finite difference method. Graphs are presented to explore the underlying physics of the problem.

2. Mathematical Formulation

Here we consider the steady three-dimensional incompressible boundary layer flow of nanofluid over an exponentially stretched sheet in two lateral directions. The sheet is located at z = 0 and the flow is restricted to $z \geq 0$. The velocities of the sheet along x—and y—directions respectively are given by $U_w(x,y) = U_0 e^{\frac{x+y}{L}}$ and $V_w(x,y) = V_0 e^{\frac{x+y}{L}}$. The temperature of the sheet is maintained at $T_w(x,y) = T_\infty + T_0 e^{\frac{A(x+y)}{2L}}$ While $C_w(x,y) = C_\infty + C_0 e^{\frac{A(x+y)}{2L}}$ is the nanoparticle volume fraction at the sheet where T_∞ and C_∞ are the ambient values of temperature and nanoparticle volume fraction respectively. Based on the boundary layer assumptions, the governing equations for the conservations of mass, momentum, energy and nanoparticles mass are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=v\frac{\partial^2 u}{\partial z^2} \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=v\frac{\partial^2 v}{\partial z^2} \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2}+\tau\left[D_B\left(\frac{\partial C}{\partial Z}\frac{\partial T}{\partial Z}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial Z}\right)^2\right] \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\left[D_B\left(\frac{\partial^2 C}{\partial Z^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial Z^2}\right)\right]-K_r^2(\mathrm{C}-C_\infty)\left(\frac{T}{T_\infty}\right)^n e^{\left(-\frac{E_a}{\kappa T}\right)} \tag{5}$$

Where u, v and w are the velocity components along the x-, y- and z—directions respectively, v is the kinematic viscosity, T is the fluid temperature, C is the nanoparticle volume fraction, α is the thermal diffusivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient and τ (= $(\rho c)_p/(\rho c)_f$) is the ratio of the effective heat capacity of the nanoparticle material to the effective heat capacity of the base fluid. The boundary conditions for the problem are:

$$u=U_w(x,y),\qquad v=V_w(x,y),\qquad w=0,$$
$$T=T_w(x,y),\quad C=C_w(x,y)\quad \text{at}\quad Z=0$$
$$u=0,T\to T_\infty,C\to C_\infty\ \text{as}\ z\to\infty \tag{6}$$

By using the following dimensionless variables

$$u=U_0e^{\frac{x+y}{L}}f',\qquad v=U_0e^{\frac{x+y}{L}}g',$$
$$w=-\sqrt{\frac{vU_0}{2L}}e^{\frac{x+y}{2L}}\left(f+\eta f'+g+\eta g'\right),$$
$$T=T_\infty+T_0e^{\frac{A(x+y)}{2L}}\theta,$$
$$C=C_\infty+C_0e^{\frac{A(x+y)}{2L}}\phi,\qquad \eta=\sqrt{\frac{U_0}{2vL}}e^{\frac{x+y}{2L}}z, \tag{7}$$

Equation (1) is identically satisfied and Equations (2)−(7)take the following forms

$$f''-2(f'+g')f'+(f+g)f''=0, \tag{8}$$

$$g''-2(f'+g')g'+(f+g)g''=0, \tag{9}$$

$$\frac{1}{\mathrm{Pr}}\theta''-A(f'+g')\theta+(f+g)\theta'+N_b\phi'\theta'+N_t\theta'=0 \tag{10}$$

$$\phi''-ScA(f'+g')\phi+Sc(f+g)\phi'+\frac{N_t}{N_b}\theta''-Le\sigma(1+\delta\theta)^n\phi e^{\left(-\frac{E}{1+\delta\theta}\right)}=0, \tag{11}$$

$$f(0)=g(0)=0,\ f'(0)=1,\ g'(0)=\lambda,\ \theta(0)=1$$
$$,\phi(0)=1,\ f'(+\infty)\to0,\ g'(+\infty)\to0,$$
$$\theta(+\infty)\to0,\ \phi(+\infty)\to0 \tag{12}$$

where $\lambda = V_0/U_0$ is the velocity ratio, Nb = $\tau D_B(C_w - C_\infty)/\nu$ is the Brownian motion parameter,Nt = $\tau D_T(T_w - T_\infty)/T_\infty\ \nu$ is the thermophoresis parameter, Pr = ν/α is the Prandtl number,Sc = ν/D_B is the Schmidt number.

3. Numerical Method

The nonlinear coupled differential equations (8)–(11) along with the boundary condition (12) are solved numerically by using finite difference method. The effects of binary chemical reaction and activation energy on steady three-dimensional incompressible boundary layer flow of nanofluid over an exponentially stretched sheet are analyzed numerically and graphically.

4. Results and Discussion

On investigation, the role of all the non-dimensional parameters such as magnetic parameter (M), Brownian motion parameter(Nb), Thermophoresis parameter (Nt), temperature difference parameter δ,fitted rate constant n, reaction rate σ respectively. The influence of the pertinent physical parameters on the concentration profiles of the nanofluid can be observed from the graphical results.

Fig.1 represents the effects of magnetic parameter (M) on the dimensionless concentration profiles. It shows that concentration field decreases with an increasing values of Magnetic parameter (M) Fig.2 and Fig.3 represents the effects of values Brownian motion parameter(Nb) and Thermophoresis parameter (Nt) on concentration profile has an opposite effect in the values of Brownian motion parameter(Nb) and Thermophoresis parameter (Nt). Fig.4 represents effects of values temperature difference parameter on concentration profile decreases with the increasing value of temperature difference parameter δ. Similarly in Fig.5, concentration profile decreases with the increasing value of fitted rate constant *n*. Fig.6 represents Effects of values reaction rate σ on concentration profiles. It shows that concentration field increases with an increasing values of reaction rate σ .

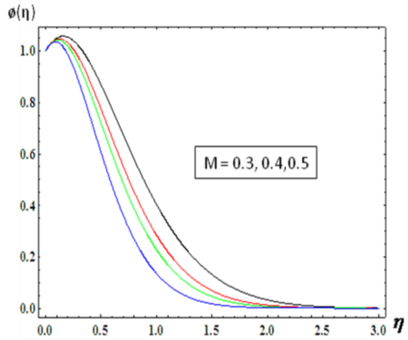

Fig 1 : Effect of Magnetic parametre on concentration profile for the values of $Pr=Sc=5, \lambda=0.5, A=3, Nb=1, Nt=1$

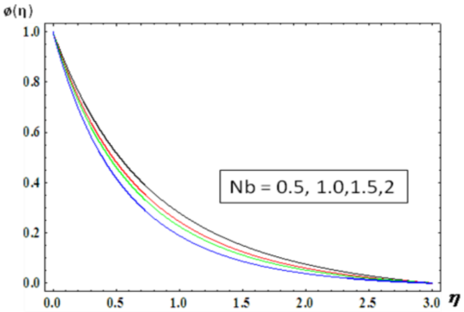

Fig 2 : Effect of Brownian motion parameter on concentration profile for the values of $Pr=1, \lambda=0.5, A=3, M=1$

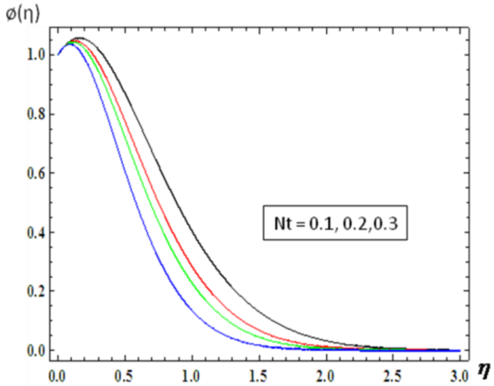

Fig 3 : Effect of thermophoresis parameter on concentration profile for the values of $Pr=Sc=5, \lambda=0.5, A=3$

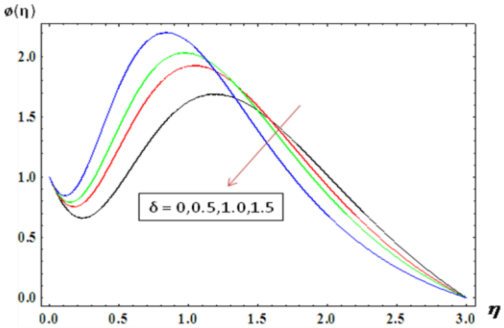

Fig 4 : Effect of temperature difference parameter on concentration profile for the values of $Nb=Nt=0.1, Pr=Sc=5$

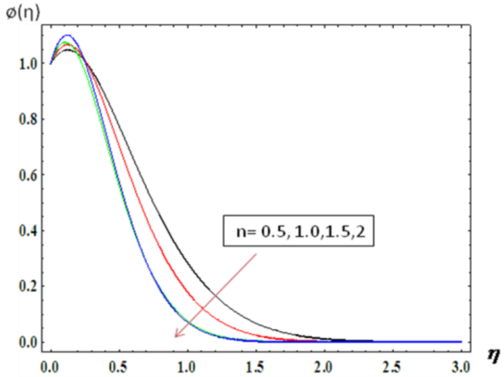

Fig 5 : Effect of fitted rate contant on concentration profile for the values of $Nb=Nt=0.1, \lambda=0.5, A=3$

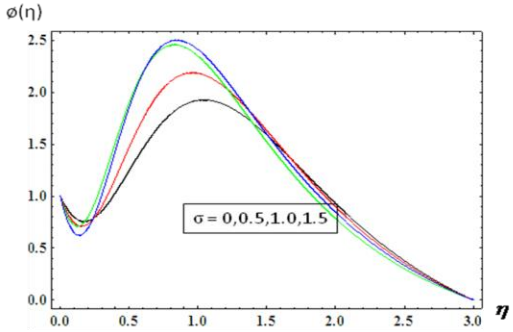

Fig 6 : Effect of reation rate σ on ϕ for the values of Nb=Nt=0.1,Pr=Sc=5

5. Conclusion

In this study, a mathematical model for steady three-dimensional incompressible boundary layer flow of nanofluid over an exponentially stretched sheet has been designed considering various physical parameters such as effects of radiation, chemical reaction, binary chemical reaction and activation energy. The validity of the present computations has been confirmed via benchmarking based on several earlier studies.

- The increase in magnetic parameter is to decrease in concentration profile.
- The increase in temperature difference parameter δ is to decrease in concentration profile.
- The increase of fitted rate constant *n* is to decreases concentration profile.
- The increasing values reaction rate σ increases concentration profile.
- Brownian motion and thermophoresis parameters decrease the concentration profile.

References

[1] Sakiadis BC (1961) Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow. AIChE Journal, 7, 26–28.
[2] Crane LJ (1970) Flow past a stretching plate. Z Angew Math Phys, 21, 645–647.
[3] Rajagopal KR, Na TY, Gupta AS (1984) Flow of a viscoelastic fluid over a stretching sheet. Rheol Acta,23, 213–215.
[4] Lawrence PS, Rao BN (1992) Heat transfer in the flow of a viscoelastic fluid over a stretching sheet.
[5] Acta Mech, 93, 53–61.
[6] Grubka LJ, Bobba KM (1985) Heat transfer characteristics of a continuous, stretching surface with variable temperature. J Heat Trans, 107, 248–250.
[7] Banks WHH (1983) Similarity solutions of the boundary-layer equations for a stretching wall. J Méc
[8] Theor Appl, 2, 375–392.
[9] Chen CK, Char MI (1988) Heat transfer of a continuous stretching surface with suction or blowing. J
[10] Math Anal Appl, 135, 568–580.
[11] AliME (1995) On thermal boundary layer on a power-law stretched surface with suction or injection. Int J Heat Mass Transf, 16, 280–290.
[12] Pop I, Na TY (1998) A note on MHD flow over a stretching permeable surface. Mech Res Comm, 25, 263–269.
[13] Magyari E, Keller B (2000) Exact solutions for self-similar boundary-layer flows induced by permeable stretching walls. Eur J Mech B-Fluids, 19, 109–122.
[14] Liao SJ (2003) On the analytic solution of magnetohydrodynamic flows of non-Newtonian fluids over a stretching sheet. J Fluid Mech, 488, 189–212.
[15] Liao SJ (2006) An analytic solution of unsteady boundary-layer flows caused by an impulsively stretching plate. Commun Nonlinear Sci Numer Simulat, 11, 326–339.
[16] Xu H, Liao SJ, Pop I (2007) Series solutions of unsteady three-dimensional MHD flow and heat transfer in the boundary layer over an impulsively stretching plate. Eur J Mech B-Fluids, 26, 15–27.
[17] Sajid M, Hayat T, Pop I (2008) Three-dimensional flow over a stretching surface in a viscoelastic fluid. Nonlinear Anal: Real World Appl, 9, 1811–1822.
[18] Liu IC, Anderson HI (2008) Heat transfer over a bidirectional stretching sheet with variable thermal conditions. Int J Heat Mass Transf, 51, 4018–4024.

[19] Xu H, Liao SJ (2009) Laminar flow and heat transfer in the boundary-layer of non-Newtonian fluids over a stretching flat sheet. Comp Math Appl, 54, 1425–1431.

[20] Hayat T, Mustafa M, Sajid M (2011) on mass transfer in three-dimensional flow of a viscoelastic fluid. Num Meth Partial Diff Eq, 27, 915–936.

[21] Khan JA, Mustafa M, Hayat T, Farooq MA, Alsaedi A, Liao SJ (2014) On model for three-dimensional flow of nanofluid: An application to solar energy. J Molec Liqu, 194, 41–47.

[22] Magyari E, Keller B (1999) Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. JPhy D: Appl Phy, 32, 577–585.

[23] Khan SK, Sanjayanand E (2005) Viscoelastic boundary layer flow and heat transfer over an exponential stretching sheet. Int J Heat Mass Transf, 48,1534–1542.

[24] Sajid M, Hayat T (2008) Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. Int Commun Heat Mass Transf, 35, 347–356.

[25] Liu IC, Wang HH, Peng YF (2013) Flow and heat transfer for three-dimensional flowover an exponentially stretching surface. Chem Eng Comm, 200, 253–268.

[26] Nadeem S, Zaheer S, Fang T (2011) Effects of thermal radiation on the boundary layer flow of a Jeffrey fluid over an exponentially stretching surface. Numer Algor, 57, 187–205.

[27] Masuda H, Ebata A, Teramae K, Hishinuma N (1993) Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles (dispersion of c-Al2O3, SiO2 and TiO2 ultra-fine particles). NetsuBussei (in Japanese), 4, 227–233.

[28] Choi SUS, Eastman JA (1995) Enhancing thermal conductivity of fluids with nanoparticles. in: The Proceedings of the 1995 ASME International Mechanical Engineering Congress and Exposition, San Francisco, USA, ASME, FED 231/MD, 66, 99–105.

[29] Trieb F, Nitsch J (1998) Recommendations for the market introduction of solar thermal power stations. Renew Ener, 14, 17–22.

[30] Otanicar TP, Phelan PE, Prasher RS, Rosengarten G, Taylor RA (2010) Nanofluid-based direct absorption solar collector. J Renew Sustain Ener, 2,033102.

[31] Ladjevardi SM, Asnaghi A, Izadkhast PS, Kashani AH (2013) Applicability of graphite nanofluids in direct solar energy absorption. Solar Energy, 94, 327–334

[32] Ebaid A, Aly EH (2013) Exact analytical solution of peristaltic nanofluids flow in an asymmetric channel with flexible walls and slip condition: Application to the cancer treatment. Comput Math Methods Med.

[33] Buongiorno J (2006) Convective transport in nanofluids. ASME J Heat Transf, 128, 240–250.

[34] Kuznetsov AV, Nield DA (2010) Natural convective boundary-layer flow of a nanofluid past a vertical plate. Int J Therm Sci, 49, 243–247.

[35] Nield DA, Kuznetsov AV (2009) The Cheng—Minkowycz problem for natural convective boundarylayer flow in a porous medium saturated by a nanofluid. Int J Heat Mass Transf, 52, 5792–5795.

[36] Khan WA, Pop I (2010) Boundary-layer flow of a nanofluid past a stretching sheet. Int J Heat Mass

[37] Transf, 53, 2477–2483.

[38] Makinde OD, Aziz A(2011) Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. Int J Therm Sci, 50, 1326–1332.

[39] Mustafa M, Hayat T, Pop I, Asghar S, Obadiat S (2011) Stagnation-point flow of a nanofluid towards a stretching sheet. Int J Heat Mass Transf, 54, 5588–5594.

[40] Mustafa M, Farooq MA, Hayat T, Alsaedi A (2013) Numerical and series solutions for stagnation-point

[41] flow of nanofluid over an exponentially stretching sheet.PLoS ONE, 8.

[42] Mustafa M, Hayat T, Obaidat S (2013) Boundary layer flow of a nanofluid over an exponentially stretching sheet with convective boundary conditions.Int J Num Meth Heat & Fluid Flow, 23, 945–959.

[43] Rana P, Bhargava R (2012) Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: A numerical study. Comm Nonlinear Sci Num Simul, 17, 212–226.

[44] Bég OA, Khan MS, Karim I, Alam MM, Ferdows M (2013) Explicit numerical study of unsteady hydromagnetic mixed convective nanofluid flow from an exponentially stretching sheet in porous media. Appl Nanosci.

[45] Rasekh A, Ganji DD, Tavakoli S (2012) Numerical solutions for a nanofluid past over a stretching circular cylinder with non-uniform heat source. Front Heat Mass Transf,http://dx.doi.org/10.5098/hmt.v3.4.3003.

[46] Uddin MJ, Khan WA, Ismail AI (2012) MHD free convective boundary layer flow of a nanofluid past a flat vertical plate with Newtonian heating boundary condition. PLoS One, 7.

[47] Ashorynejad HR, Sheikholeslami M, Pop I, Ganji DD (2013) Nanofluid flow and heat transfer due to a stretching cylinder in the presence of magnetic field. Heat Mass Tranf, 49, 427–436.

[48] Mustafa M, Hayat T Alsaedi A(2013).Unsteady boundary layer flow of nanofluid past an impulsively stretching sheet.J Mech, 29, 423–432.

[49] Turkyilmazoglu M (2013) Unsteady convection flow of some nanofluids past a moving vertical flat plate with heat transfer. J Heat Transf Trans ASME, 136, 031704.

[50] Mushtaq A, Mustafa M, Hayat T, Alsaedi A (2014) Nonlinear radiative heat transfer in the flow of nanofluid due to solar energy: A numerical study. J Taiwan Inst Chem Eng, 45, 1176–1183.

[51] Turkyilmazoglu M (2014) Nanofluid flow and heat transfer due to a rotating disk. Comp Fluids, 94,

[52] 139–146.

[53] Sheikholeslami M, Bandpy MG, Ellahi R, Hassan M, Soleimani S (2014) Effects of MHD on Cu—water nanofluid flow and heat transfer by means of CVFEM. J Magn Magn Mater, 349, 188–200.

[54] Safaei MR, Togun H, Vafai K, Kazi SN, Badarudin A (2014) Investigation of heat transfer enchantment in a forward-facing contracting channel using FMWCNT nanofluids. Num Heat Transf Part A, 66, 1321–1340.

[55] Malvandi A, Ganji DD (2014) Brownian motion and thermophoresis effects on slip flow of alumina/water nanofluid inside a circularmicrochannel in the presence of a magnetic field. Int JThermSci, 84, 196–206.

[56] Malvandi A, Ganji DD (2014) Mixed convective heat transfer of water/alumina nanofluid inside a vertical microchannel. Powder Technol, 263, 37–44.

[57] Malvandi A, Ganji DD (2014) Effects of nanoparticle migration on force convection of alumina/water nanofluid in a cooled parallel-plate channel. Adv Powd Technol, 25,1369–1375.

[58] Karimipour A, Esfe MH, Safaei MR, Semiromi DT, Jafari S, Kazi SN (2014) Mixed convection of Copper- Water nanofluid in a shallow inclined lid driven cavity using lattice Boltzmann method. Physica A,402,150–168.

[59] Sadeqi S, Khabazi N, Sadeghy K (2011) Blasius flow of thixotropic fluids: A numerical study. Commun Nonlinear Sci Numer Simul, 16, 711–721.

[60] Javed T, Ali N, Abbas Z, Sajid M (2013) Flow of an Eyring-Powell non-Newtonian fluid over a stretching sheet. Chem Eng Commun, 200, 327–336.

[61] Kumar H, (2013).Heat transfer in MHD boundary-layer flow through a porous medium, due to a nonisothermal stretching sheet, with suction, radiation, and heat annihilation. Chem Eng Commun, 200,

[62] 895–906.

[63] IbrahimW, Shankar B (2013) MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions. Comp Fluids, 75, 1–10.

[64] Ibrahim W, Shankar B (2014) Magnetohydrodynamic boundary layer flow and heat transfer of a nanofluid over non-isothermal stretching sheet. J Heat Transf, 136, 051701.

[65] Makinde OD, Khan WA, Khan ZH (2013) Buoyancy effects on MHD stagnation point flow and heat transfer of a nanofluid past a convectively heated stretching/shrinking sheet. Int J Heat Mass Transf, 62, 526–533.

[66] Abbas Z, Javed T, Ali N, Sajid M (2014) Flow and heat transfer of Maxwell fluid over an exponentially stretching sheet: A non-similar solution. Heat Transf Asian Res, 43, 233–242.

[67] Cebeci T, Bradshaw P (1988) Physical and computational aspects of convective heat transfer. Springer-Verlag, New York, Chapter 13.

[68] Abramowitz M, Stegun IA (1965) Handbook of mathematical functions. Dover, New York.

[69] Maïga SEB, Nguyen CT, Galanis N, Roy G,Maré T, CoqueuxM (2006) Heat transfer enhancement in turbulent tube flow using Al2O3 nanoparticle suspension. Int J Num Meth Heat & Fluid Flow, 16, 275–292.